Name: ______________________

MATH 250 Instructor: ______________________

Spring 2022 Section: ______________________

Exam 1 - Sample

This exam has 9 questions for a total of 100 points.

Check that your exam has all 9 questions. In order to obtain full credit, all work must be shown.

Only Real Valued solution are accepted.

Notes, devices, communication with other people is NOT permitted (except to ask an instructor a question).

Problem	**Points**	**Your score**
1	10	
2	10	
3	10	
4	10	
5	15	
6	10	
7	15	
8	10	
9	10	
Total	100	

1. (10 points) Each question below is worth 2 points. For a), b), c), d) and e) write "True" if the statement is true, or "False" if the statement is false. You must write **the full word True or False** to receive credit.

a) If y_1 and y_2 are solutions to $y'' + 15t^2y' - e^t y = 0$, then
$2y_1 - 3y_2$ must also be a solution ____________

b) The ODE $y'' + (2 - e^t)(t + y) = 0$ is linear and homogeneous ____________

c) The ODE $\dfrac{d^2 f}{dx^2} + e^2 \dfrac{df}{dx} = f$ is linear ____________

d) $y(t) = 1$ is *the only* constant solution to
$y = (y'')^2 + (y')^2 + y^2$ ____________

e) $y(t) = t$ is a solution to $ty'' - y' + ty = t^2$ ____________

2. (10 points) Use the theorem on *existence and uniqueness* of solutions to ODE's to determine the largest interval on which a solution is guaranteed to exist for each ODE below:

a) $2y' + \dfrac{y}{\ln t} = e^{-t} \qquad y(e) = 12$

b) $y' - \dfrac{y}{\arctan t} = 1 \qquad y(-1) = -3$

3. (10 points) Find the solution to the initial value problem (assume $t > 0$)

$$y' + \frac{y}{2\sqrt{t}} = 1 \qquad y(1) = 1$$

4. (10 points) Find an explicit solution to the initial value problem

$$y' = e^{t-y} \qquad y(0) = \ln(3)$$

5. (15 points) Consider the ODE:

$$\frac{dr}{d\theta} = \frac{r^2\theta}{(\theta^2 + 1)}$$

a) List all constant solutions (if any)

b) Solve the ODE

6. (10 points) For the ODE :

$$y' = \sin(y) - 1$$

a) Graph y' vs. y, and draw the phase line

b) Determine the critical points (equilibrium or constant solutions) if any, and classify their stability.

c) Graph the solution which satisfies the initial condition $y(\pi) = \dfrac{\pi}{4}$.

If you draw several curves, **label the one curve** which satisfies the initial condition. If the graph has asymptotes, label the asymptotes as well.

7. (15 points) For the ODE below, assume that $k > 0$ is a constant.

$$\frac{dy}{dt} = k - e^{-y}$$

a) Determine all critical points (constant or equilibrium solutions) in terms of k and classify their stability

b) Suppose a solution of the ODE $y(t)$, satisfies the initial condition $y(0) = 0$. For what range of values of k would this solution be an increasing function?

c) Suppose now $k = 1$ in the ODE above, and a certain solution of the ODE satisfies the initial condition $y(0) = 0$. Is this solution increasing, decreasing, or constant?

8. (10 points) Solve the IVP below:

$$y'' - 6y' + 9y = 0 \qquad y(0) = 0,\ y'(0) = -2$$

9. (10 points) Solve the IVP below:

$$y'' + 4y' + (4 + \pi^2)y = 0 \qquad y(0) = 0,\ y'(0) = 3\pi$$