Name: ____________________

MATH 250 Student Number: ____________________

Spring 2022 Instructor: ____________________

Exam 2 - Sample Section: ____________________

This exam has 7 questions for a total of 100 points.

Only Real Valued solution are accepted.

1. (15 points) Find the general solution to the ODE

$$y'' - 3y' - 4y = 4\cos(t) - 9$$

2. (10 points) Determine a suitable form for the particular solution of the ODE when using the method of undetermined coefficients. (You do not need to solve for the coefficients.)

$$y'' + y' + y = 8te^t + 8t\sin(t)$$

3. (20 points) The position (measured in meters) of a mass tied to a spring hanging from the ceiling is given by

$$u(t) = -e^{-t}\cos t - e^{-t}\sin t$$

in which positive values of u represent positions below the equilibrium position ($u = 0$). Assume the mass begins moving at time $t = 0$.

a) Where was the mass when it began moving?

b) How fast was the mass going, and in which direction, when it began moving?

c) When does the mass hit its *lowest* point?

d) When does the mass hit its *next lowest* point?

4. (20 points) The position of a mass tied to a spring hanging from the ceiling is given by $u(t) = 3\sin(2t)$ (measured in meters), in which positive values of u represent positions below the equilibrium position ($u = 0$). Assume the mass begins moving at time $t = 0$.

a) What is the period of motion?

b) At $t = 0$, the mass is at the equilibrium position, with an initial downward velocity. When will the mass first pass through the equilibrium position *going up*?

c) List all of the times the mass passes through equilibrium *going down*.

d) What is the amplitude of oscillation, R, for this mass and what does it mean physically?

e) When does the mass hit its lowest point?

5. (10 points) Compute

a) $\mathcal{L}\{e^{-2t}\cos(3t) - 4\}$

b) $\mathcal{L}^{-1}\left\{\frac{1}{s+2} - \frac{2s-1}{s^2+2}\right\}$

6. (15 points) Use Laplace transforms to solve the IVP.

   You must Laplace transforms to receive full credit.

$$y'' + 4y' + 5y = \sin(t) \qquad y(0) = 1,\ y'(0) = 0$$

7. (10 points) Each question below is worth 2 points.

a) Suppose the position of a mass tied to a spring hanging from the ceiling is given by the function $u(t)$ (in which the value $u = 0$ represents the "equilibrium position") modeled by the ODE

$$u'' + 2u' + 2u = 0$$

Once set in motion, will this mass oscillate infinitely many times, going up and down to the same height and depth each time? ____________

b) Suppose the position of a mass tied to a spring hanging from the ceiling is given by the function $u(t)$ (in which the value $u = 0$ represents the "equilibrium position"), modeled by the ODE

$$u'' + 4u' + ku = 0$$

For what values of k will the mass oscillate infinitely many times? ____________

c) Suppose the position of a mass tied to a spring hanging from the ceiling is given by the function $u(t)$ (in which the value $u = 0$ represents the "equilibrium position"), modeled by the ODE

$$u'' + ku = 0$$

For what values of k will the limit $\lim_{t \to \infty} u = 0$? ____________

d) Suppose the position of a mass tied to a spring hanging from the ceiling is given by the function $u(t)$ (in which the value $u = 0$ represents the "equilibrium position"), modeled by the ODE

$$u'' + \gamma u' + 25u = 0$$

For what values of γ will the mass oscillate infinitely many times, going up and down less high and low each time? ____________

e) Compute $\mathcal{L}^{-1}\left\{\frac{1}{s^3}\right\}$ ____________

TABLE 6.2.1 **Elementary Laplace Transforms**

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$	Notes
1. 1	$\dfrac{1}{s}, \quad s > 0$	Sec. 6.1; Ex. 4
2. e^{at}	$\dfrac{1}{s-a}, \quad s > a$	Sec. 6.1; Ex. 5
3. $t^n, \quad n$ a positive integer	$\dfrac{n!}{s^{n+1}}, \quad s > 0$	Sec. 6.1; Prob. 24
4. $t^p, \quad p > -1$	$\dfrac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$	Sec. 6.1; Prob. 24
5. $\sin(at)$	$\dfrac{a}{s^2+a^2}, \quad s > 0$	Sec. 6.1; Ex. 7
6. $\cos(at)$	$\dfrac{s}{s^2+a^2}, \quad s > 0$	Sec. 6.1; Prob. 5
7. $\sinh(at)$	$\dfrac{a}{s^2-a^2}, \quad s > \lvert a\rvert$	Sec. 6.1; Prob. 7
8. $\cosh(at)$	$\dfrac{s}{s^2-a^2}, \quad s > \lvert a\rvert$	Sec. 6.1; Prob. 6
9. $e^{at}\sin(bt)$	$\dfrac{b}{(s-a)^2+b^2}, \quad s > a$	Sec. 6.1; Prob. 10
10. $e^{at}\cos(bt)$	$\dfrac{s-a}{(s-a)^2+b^2}, \quad s > a$	Sec. 6.1; Prob. 11
11. $t^n e^{at}, \quad n$ a positive integer	$\dfrac{n!}{(s-a)^{n+1}}, \quad s > a$	Sec. 6.1; Prob. 14
12. $u_c(t) = \begin{cases} 0 & t < c \\ 1 & t \geq c \end{cases}$	$\dfrac{e^{-cs}}{s}, \quad s > 0$	Sec. 6.3
13. $u_c(t) f(t-c)$	$e^{-cs}F(s)$	Sec. 6.3
14. $e^{ct} f(t)$	$F(s-c)$	Sec. 6.3
15. $f(ct)$	$\dfrac{1}{c}F\left(\dfrac{s}{c}\right), \quad c > 0$	Sec. 6.3; Prob. 17
16. $(f * g)(t) = \int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$	Sec. 6.6
17. $\delta(t-c)$	e^{-cs}	Sec. 6.5
18. $f^{(n)}(t)$	$s^n F(s) - s^{n-1} f(0) - \cdots - f^{(n-1)}(0)$	Sec. 6.2; Cor. 6.2.2
19. $(-t)^n f(t)$	$F^{(n)}(s)$	Sec. 6.2; Prob. 21