

Name: _____

ID Number: _____

Instructions: Clearly answer each of the questions below. Remember to check the back side. Use full sentences and proper grammar for verbal answers. Show your work and any formulas you employ. Simplify all answers as far as possible. Box your answers.

1. What is the definition of $\mathcal{L}[f(t)]$?

Answer:

$$\mathcal{L}[f(t)] = \int_0^{\infty} e^{-st} f(t) dt$$

2. Find $\mathcal{L}[7e^{-5t}]$

Answer:

$$\frac{7}{s+5}$$

3. Find $\mathcal{L}[3t^2 + 1]$

Answer:

$$\mathcal{L}[3t^2 + 1] = 3\mathcal{L}[t^2] + \mathcal{L}[1] = 3\frac{(2!)}{s^3} + \frac{1}{s} = \frac{6}{s^3} + \frac{1}{s}$$

4. Find (and simplify) $\mathcal{L}[3 + 9t^2 + 14e^{-5t}]$

Answer: You can do this out the long way, but the better method is to use the linearity of the Laplace transform \mathcal{L} , and the answers of (2) and (3), so

$$\begin{aligned} \mathcal{L}[3 + 9t^2 + 14e^{-5t}] &= 3\mathcal{L}[1 + 3t^2] + 2\mathcal{L}[7e^{-5t}] \\ &= 3\left(\frac{6}{s^3} + \frac{1}{s}\right) + 2\left(\frac{7}{s+5}\right) \\ &= \frac{18}{s^3} + \frac{3}{s} + \frac{14}{s+5} \end{aligned}$$

5. If $\ddot{y} - 5\dot{y} + 6y = te^{3t} + \cos(5t)$ when $y(0) = 4$ and $y'(0) = 9$, find $\mathcal{L}[y(t)]$.

Answer:

$$\mathcal{L}[y] = \frac{1}{s^2 - 5s + 6} \left(4s - 11 + \frac{1}{(s-3)^2} + \frac{s}{s^2 + 25} \right)$$

6. If $\mathcal{L}[y] = \frac{3-s}{s^2-9s}$, find $y(t)$?

Answer:

$$\begin{aligned} \frac{3-s}{s^2-9s} &= -\frac{1}{3} \left(\frac{1}{s} \right) - \frac{2}{3} \left(\frac{1}{s-9} \right) \\ \mathcal{L}^{-1} \left(\frac{3-s}{s^2-9s} \right) &= \frac{-1}{3} - \frac{2}{3} e^{9t} \end{aligned}$$

7. If $\mathcal{L}[y] = \frac{2-5s}{s^2+9}$, find $y(t)$?

Answer:

$$\begin{aligned} \frac{2-5s}{s^2+9} &= \frac{2}{3} \frac{3}{s^2+9} - 5 \frac{s}{s^2+9} \\ y(t) &= \frac{2}{3} \sin(3t) - 5 \cos(3t) \end{aligned}$$