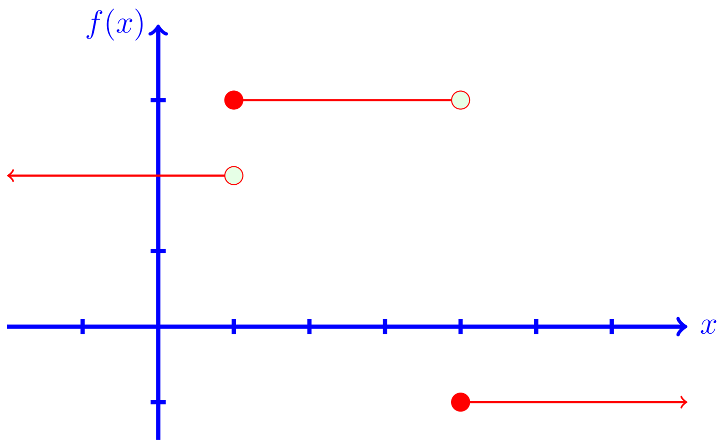
Name:______________________________ Math 250, Section 7, Quiz 5

Instructions: Clearly answer each of the questions below. Remember to check the back side. Use full sentences and proper grammer for verbal answers. Show your work and any formulas you employ. Simplify all answers as far as possible. Box your answers.

1. Plot the function $f(x) = 2 + u_1(x) - 4u_4(x)$.



2. A dose of medicine is given to a patient and the blood concentration obeys $y'' + 5y' + 4y = 2\delta(t-7)$. Find $\mathscr{L}[y(t)]$ when $y(0) = 2$, $y'(0) = 3$.

$$(s^2\mathscr{L}[y] - 2s - 3) + 5(s\mathscr{L}[y] - 2) + 4\mathscr{L}[y] = 2e^{-7s}$$
$$\mathscr{L}[y] = \frac{2e^{-7s} + 13 + 2s}{s^2 + 5s + 4}$$

3. Find $y(t)$ when $Y(s) = (e^{-5s} - e^{5s})(s+4)/((s+4)^2 + 9)$.

$$y(t) = u_5(t)e^{-4(t-5)}\cos(3(t-5)) - u_{-5}(t)e^{-4(t+5)}\cos(3(t+5))$$

4. Rewrite $y'' + 5y' + 4y = 2\delta(t-3)$ as a system of two first-order differential equations. (Be sure to clearly define your new variables.)

Let $u(t) = y(t)$ and $v(t) = y'(t)$, so

$$u' = v, \quad v' = -4u - 5v + 2\delta(t-3)$$

5. Circle all of the following that are solutions of the first-order 2x2 ODE system

$$\frac{d}{dt}\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 16 & 0 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}.$$

You can complete this problem by checking the answers rather than directly solving.

$$e^{3t}\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \quad e^{4t}\begin{bmatrix} 1 \\ 4 \end{bmatrix}, \quad 7e^{4t}\begin{bmatrix} 1 \\ 4 \end{bmatrix} - e^{-4t}\begin{bmatrix} 1 \\ -4 \end{bmatrix}, \quad e^{4t}\begin{bmatrix} 1 \\ 4 \end{bmatrix} + 9e^{-t}\begin{bmatrix} 1 \\ 4 \end{bmatrix},$$

$$e^{-4t}\begin{bmatrix} 1 \\ -4 \end{bmatrix}, \quad e^{-t}\begin{bmatrix} 1 \\ 4 \end{bmatrix}, \quad 2e^{-4t}\begin{bmatrix} 1 \\ -4 \end{bmatrix} + e^{3t}\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \quad C_1e^{-t}\begin{bmatrix} 1 \\ 4 \end{bmatrix} + C_2e^{3t}\begin{bmatrix} 4 \\ 1 \end{bmatrix},$$

The important thing to recognize is that the the principle of superposition applies -- this is a homogeneous equation, so we can add any multiples of two solutions together to get a new solution. Also, there can not be more than two "different" solutions, so once we've found two, we can rule out the rest.

$$e^{3t}\begin{bmatrix} 12 \\ 3 \end{bmatrix} = 3\left(e^{3t}\begin{bmatrix} 4 \\ 1 \end{bmatrix}\right) = \left(e^{3t}\begin{bmatrix} 4 \\ 1 \end{bmatrix}\right)' \neq \begin{bmatrix} 0 & 1 \\ 16 & 0 \end{bmatrix}\left(e^{3t}\begin{bmatrix} 4 \\ 1 \end{bmatrix}\right) = e^{3t}\begin{bmatrix} 1 \\ 64 \end{bmatrix}$$

$$e^{4t}\begin{bmatrix} 4 \\ 16 \end{bmatrix} = 4\left(e^{4t}\begin{bmatrix} 1 \\ 4 \end{bmatrix}\right) = \left(e^{4t}\begin{bmatrix} 1 \\ 4 \end{bmatrix}\right)' = \begin{bmatrix} 0 & 1 \\ 16 & 0 \end{bmatrix}\left(e^{4t}\begin{bmatrix} 1 \\ 4 \end{bmatrix}\right) = e^{4t}\begin{bmatrix} 4 \\ 16 \end{bmatrix}$$

$$e^{-4t}\begin{bmatrix} -4 \\ 16 \end{bmatrix} = -4\left(e^{-4t}\begin{bmatrix} 1 \\ -4 \end{bmatrix}\right) = \left(e^{-4t}\begin{bmatrix} 1 \\ -4 \end{bmatrix}\right)' = \begin{bmatrix} 0 & 1 \\ 16 & 0 \end{bmatrix}\left(e^{-4t}\begin{bmatrix} 1 \\ -4 \end{bmatrix}\right) = e^{-4t}\begin{bmatrix} -4 \\ 16 \end{bmatrix}$$

$$e^{-t}\begin{bmatrix} -1 \\ -4 \end{bmatrix} = -1\left(e^{-t}\begin{bmatrix} 1 \\ 4 \end{bmatrix}\right) = \left(e^{-t}\begin{bmatrix} 1 \\ 4 \end{bmatrix}\right)' \neq \begin{bmatrix} 0 & 1 \\ 16 & 0 \end{bmatrix}\left(e^{-t}\begin{bmatrix} 1 \\ 4 \end{bmatrix}\right) = e^{-t}\begin{bmatrix} 4 \\ 16 \end{bmatrix}$$