

Math 421 Homework assignment 2, part b

due Wednesday, September 14

1. Sketch the following parametric curves.

(a) $z(t) = 3 - t + (4 + t)i, t \in \mathbb{R}$.

(b) $z(t) = 2\pi t + i + e^{-2\pi it}, t \in [-2, 2]$.

(c) $(1 - it)/(1 + t^2), t \in \mathbb{R}$.

2. Suppose a wheel of radius $1/2$ meter rolls around the inside of a pipe with radius 1 meter, without slipping. Trace the path of a point on the perimeter of the wheel.

3. Sketch the following sets of points.

(a) $|z + i| = |z - i|$

(b) $|z + 2| - |z + 5| = \pm 1$

(c) $\operatorname{Re}(i\bar{z}) = 4$

(d) $|z + 1| + |z - 1| = 3$

4. Find an explicit formula for the image of

$$\{z \in \mathbb{C} : \pi/3 \leq \operatorname{Arg} z \leq 5\pi/6\}$$

under $f(z) = 1/z$.

5. Find an explicit formula for the image of

$$\{z \in \mathbb{C} : |z| < 2\}$$

under $f(z) = 1/z$.

6. Draw the set $\{z \in \mathbb{C} : \operatorname{Im}(z) \geq 5 \text{ and } \operatorname{Re}(z) \leq -1\}$ and its image under the inversion $f(z) = 1/z$, including an explicit formula for the image boundary.

7. Find an explicit formula for the image of

$$\{z \in \mathbb{C} : \pi/3 \leq \operatorname{Arg} z \leq 5\pi/6\}$$

under $f(z) = z^2$.

8. Find an explicit formula for the image of

$$\{z \in \mathbb{C} : |z| < 2\}$$

under $f(z) = z^2$.

9. Find an explicit formula for the image of

$$\{z \in \mathbb{C} : \operatorname{Re}(z) < -2\}$$

under $f(z) = \sqrt{z}$, assuming the standard branch cut along the negative real axis.

10. Prove $f(z) = z^3$ is a continuous function.