

Homework 3

Spring 2025 Math 511, Penn State UP

Introduction to asymptotics

1. Use the method of dominate balance to find as many asymptotic solutions of the following equation as you can: $1 = xy^3 + x^3y^2 + x^2y^2 - x^2y^4$.
2. Use dominate-balance techniques to find the leading-order behavior for solutions of the Airy equation $y'' - xy = 0$ as $x \rightarrow -\infty$. You will first want to compactify the domain of interest with the change of independent variable $x = -1/t$.

Bessel

3. Suppose we understand the solutions of Bessel's equation

$$x^2y'' + xy' + (x^2 - n^2)y = 0,$$

we can use those to solve other equations as well. If $y_n(x)$ is a solutions of Bessel's equation, what general set of equations can be solved by each of the following substitutions? Express you answers in wild-card form that can be easily matched against a given 2nd-order linear equation in standard form

$$y'' + a_1(x)y' + a_2(x)y = 0.$$

- (a) Equations with solution $y_n(bx)$?
- (b) Equations with solution $x^m y_n(x)$?
- (c) Equations with solution $\exp(ax)y_n(bx)$?