Problems in integrating evolution and ecology: Ising Models and the virtues of incrementalism*.

Tim

December 8, 2003

*some footnote

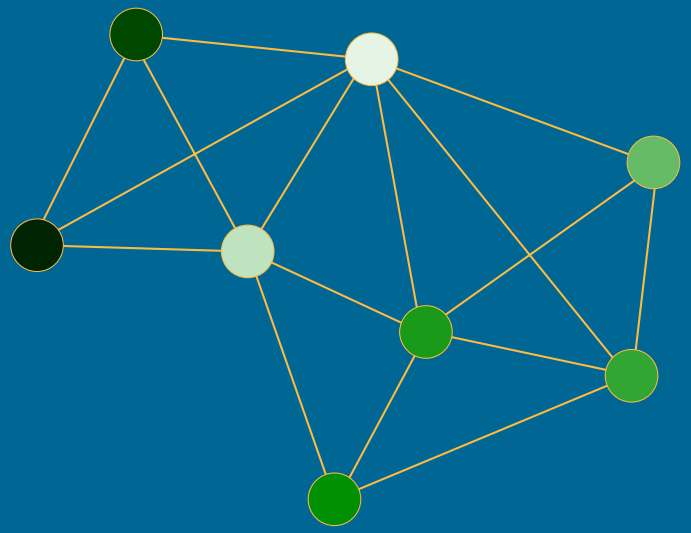
Outline

- On Tuesday, I presented several models near the interface of ecology and **evolution**, all of which earned criticism for their failures to account for the **appropriate** structures.
- Today, I'll introduce a model with more structure, but with its own problems.
  - Combinatorial landscapes
  - The Ising Model
  - Properties and classical uses
  - Extinction and Speciation on an Ising landscape
  - Problems and future work

So, how do we describe structured evolution?

One of the most powerful modern concepts in quantitative science is that of the generalized fitness landscape, introduced by Sewall Wright in 1932 [Reidys and Stadler, 2002].

A fitness landscape is a rough space described by a graph with nodes that describe fitness and edges that describe adjacency relationships.



Energy

The energy of each individual magnet is determined by its orientation relative to its neighbors and an external magnetic field:

$$E_j = -J\sigma_j \left(\sigma_{j+1} + \sigma_{j-1} + \sigma_{j+i} + \sigma_{j-i}\right) - K\sigma_j \tag{1}$$

The total energy H of a configuration is

$$H = \sum_{\forall j} \left[-J\sigma_j \left(\sigma_{j+1} + \sigma_{j-1} + \sigma_{j+i} + \sigma_{j-i}\right) - K\sigma_j\right] \tag{2}$$

This is called the Hamiltonian of the system.

Markov Chain Monte Carlo Simulation

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1. Choose a lattice site at random.

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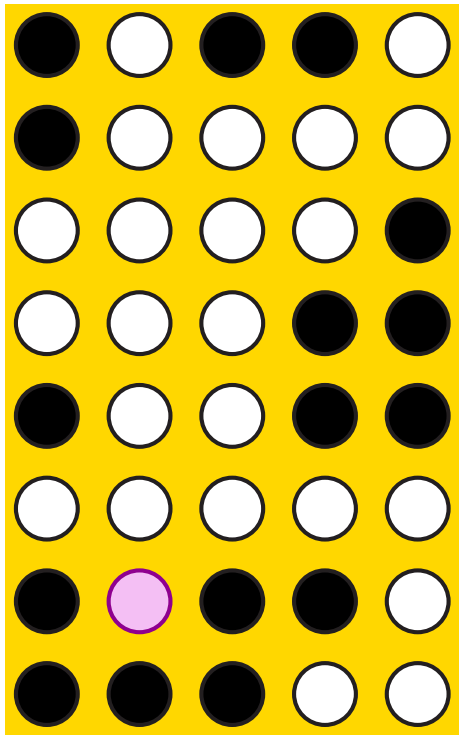
   (a) If the flip energy will be less than the current energy, flip.
   (b) If the flip energy is more than the current energy, flip with a probability proportional to the ratio of the stat energies.

An Example Time Step

Consider the Ising model with zero external field. The current energy of the purple site is



$$H_w = -J(+1)(-1-1-1+1) = 2J \ .$$

If it flips to black, it will have energy

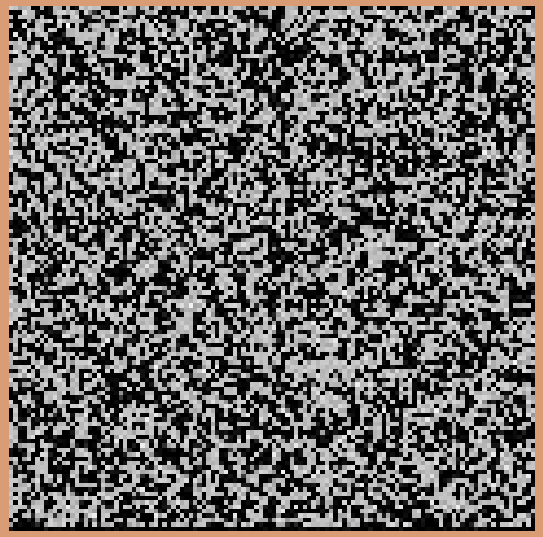
$$H_b = -J(-1)(-1-1-1+1) = -2J \ .$$

Since the energy of flipping is less than the energy of not flipping, this site will flip. If it had been black, it would flip to white with probability

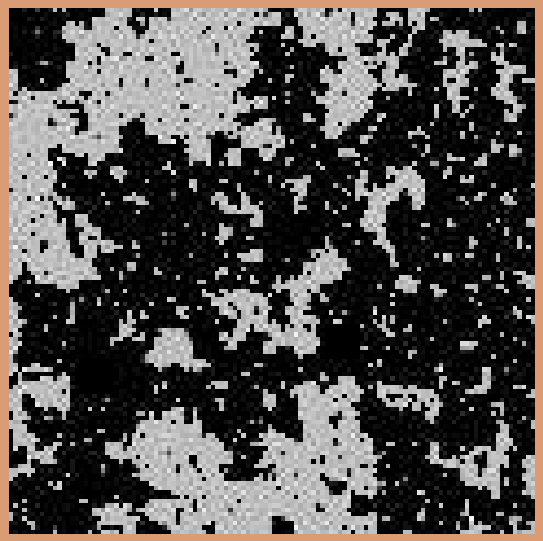
$$P = e^{-\beta(H_w - H_b)} = e^{-4J/kT} \qquad (3)$$

Snapshots of Different Temperatures

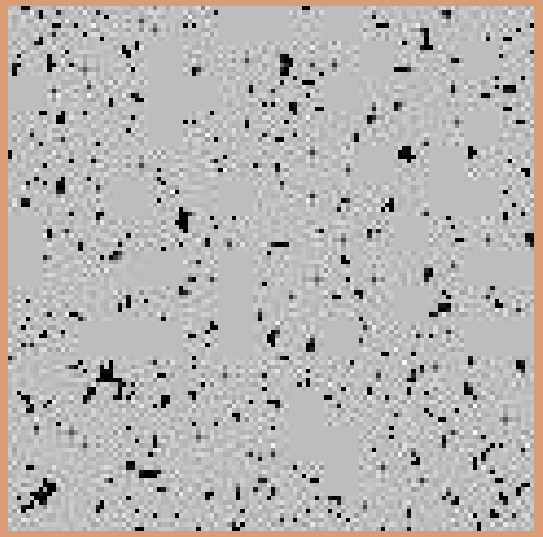
$\beta J = 0.1$



$\beta J = \sqrt{2} - 1$



$\beta J = 0.5$



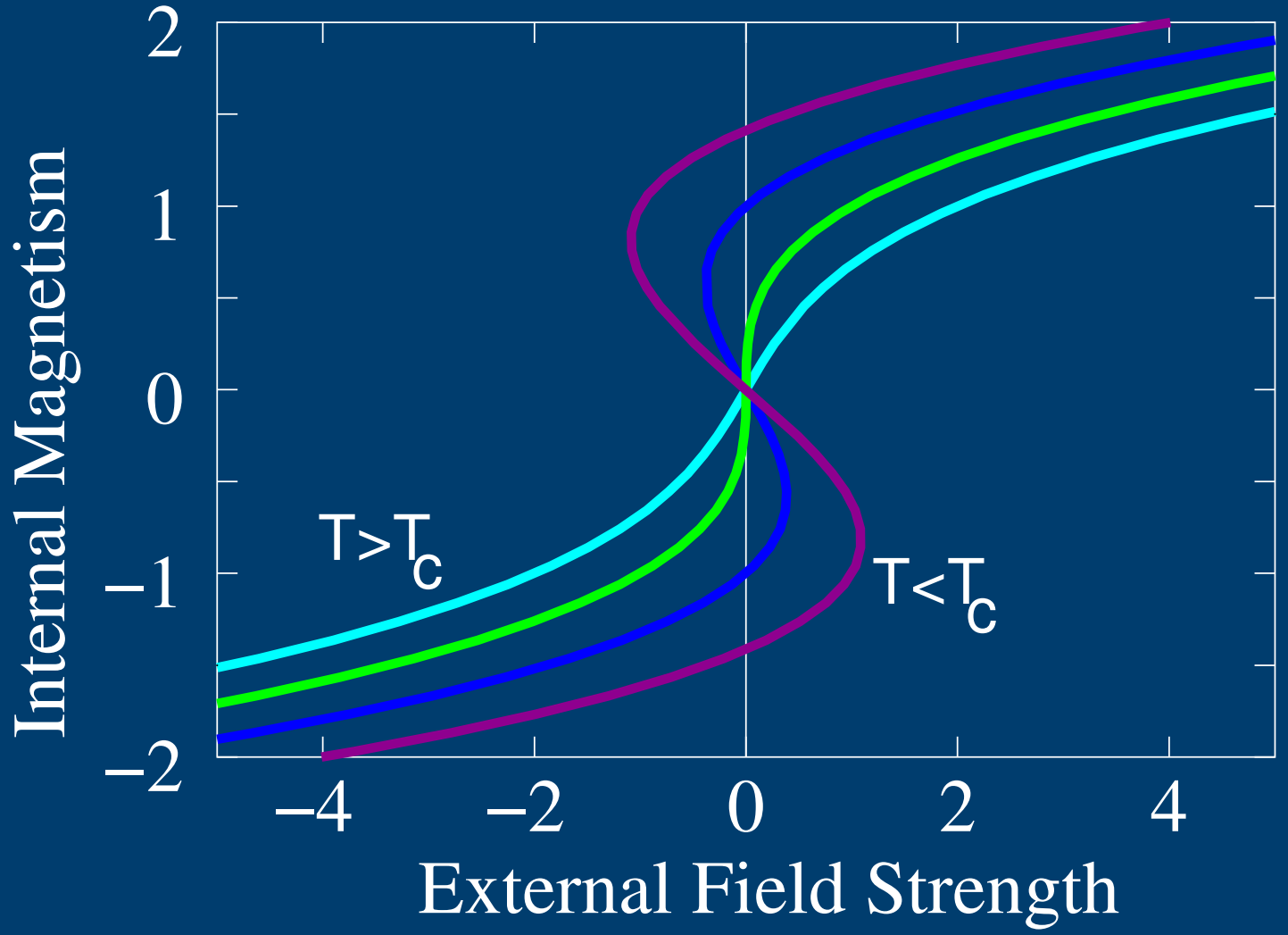
Phase Transitions in the Ising Model

The Ising model is a important because it is the simplest toy model which exhibits a phase transition in 2 and 3 dimensions. Mathematically, phase transition is a discontinuity in the macroscopic behavior of a system, and is related to bifurcations.

Melting and sublimation are commonly observed first-order phase transitions.

Iron exhibits a second order phase transition at the Curie Point of 1044 degrees Kelvin. At temperatures hotter than this, thermal fluctuations make it lose all magnetization.

Fold bifurcations



The Ising model as a fitness landscape

Now, assume that there is temporal variation in niche space, and that it can be described by an Ising model.

- Imagine a niche space, where each site can be in one of two states: Available , or Unavailable .
- The configuration of Available and Unavailable niches changes in time according to an Ising model.
- Each Available niche can be either Unoccupied or Occupied by a living organism.

References

C. M. Reidys and P. F. Stadler. Combinatorial landscapes. SIAM Review, 44(1):3–54, March 2002.

N. D. Newell. Revolutions in the history of life. Geological Society of America Special Paper, 89:63–91, 1967.